Det Kgl. Danske Videnskabernes Selskab.
Mathematisk-fysiske Meddelelser. **XIII,** 9.

RADIOMETER FORCE AND DIMENSIONS OF APPARATUS. II.

BY

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KØBENHAVN
LEVIN & MUNKSGAARD
EJNAR MUNKSGAARD
1935

Printed in Denmark.
Bianco Lunos Bogtrykkeri A/S.

Summary.

Some measurements of the radiometer force on platinum bands of various widths are described. The bands were placed in a large container; the measurements were carried out at various pressures of hydrogen and dry atmospheric air.

R_1 being the radiometer force per cm of the platinum band, and T_1 the difference in temperature between the platinum band and the container, it is shown that.

At low pressures $\frac{R_1}{T_1}$ is proportional to the width B of the band, in agreement with Professor KNUDSEN's theory and with other measurements.

At high pressures $\frac{R_1}{T_1}$ decreases rapidly with increasing B.

The maximum value of $\frac{R_1}{T_1}$ is but slightly dependent on B; but the pressure p_m at which $\frac{R_1}{T_1}$ is maximum is about inversely proportional to B.

An empirical formula of the results is given.

On a platinum band 1 cm wide placed in a narrow tube with a rectangular section radiometer forces are measured which are about 5 times greater than those on a band of the same width placed in the large container.

As a supplement to investigations previously described[1] concerning the dependency of the radiometer force on the dimensions of the apparatus, a series of measurements were made of the radiometer force on platinum bands of various widths.

The measurements were carried out with the torsion balance described by Professor KNUDSEN[2] which was pro-

[1] V. S. math.-fys. Medd. XI. 9.

[2] V. S. math.-fys. Medd. XI. 1.

vided with a thicker suspension wire of Wolfram, the torsion moment of which was found by oscillation experiments to be

$$D = 109{,}2 \frac{\text{Dyn}\cdot\text{cm}}{\text{Radian}}$$

The wire was about 100 μ thick, its length was 10,6 cm.

First, measurements were made of the radiometer force on a couple of bands of a width of $^1/_4$ cm; then these were replaced by a couple of bands $^1/_2$ cm wide, and the radiometer force on them was measured, and finally measurements were made with a pair of bands 1 cm wide.

All dimensions were carefully measured for every pair of platinum bands.

The angle of deflection was measured with a telescope and a scale with 100,0 cm scale distance.

If the radiometer force per 1 cm of the length of the band be called R_1 and the deflection α cm (reduced to angular measure), then for the three different pairs of bands

$$R = \alpha\cdot 0{,}00399 \frac{\text{Dyn}}{\text{cm}} \qquad B = 0{,}253 \text{ cm}$$
$$R = \alpha\cdot 0{,}00410 \quad » \qquad B = 0{,}502 \quad »$$
$$R = \alpha\cdot 0{,}00431 \quad » \qquad B = 1{,}001 \quad »$$

B = the mean value of the widths of the two bands. Thus, of the bands $^1/_2$ cm wide, one was 0,5036 cm and the other 0,4994 cm wide.

After having been soldered to the torsion balance, one side of the bands was blackened with platinum black; this blackening was in each case done in the same way (with the same solution of platinum chloride, the same density of current, in the same time), so as to make the black surfaces as equal as possible.

The temperature of the platinum bands was determined during the measurements by their electric resistance.

The temperature of the container was read on a thermometer placed on the glass cover.

In the following T_1 denotes the temperature difference $T - T_0$, where T is the absolute temperature of the platinum bands and T_0 that of the container, which is about room temperature.

Results of the Measurements.

The radiometer force was measured at different pressures p Dyn/cm^2 of hydrogen and of dry atmospheric air.

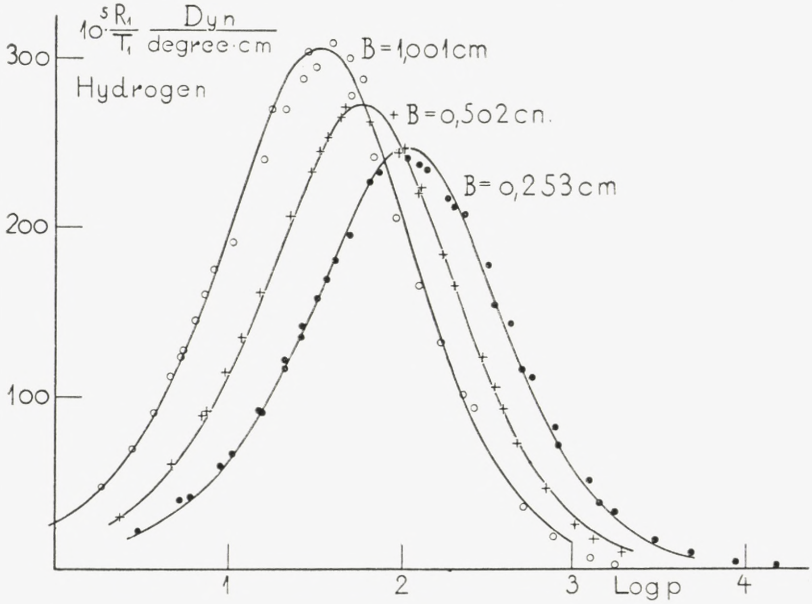
The tables below give $10^5 \cdot \frac{R_1}{T_1}$ for the pressures p and for various values of T_1, the approximate value only of T_1 being tabulated.

In figs. 1 and 2, $\frac{R_1}{T_1}$ is presented graphically as a function of log p.

The plotted values of $\frac{R_1}{T_1}$ are all valid for $T_1 = 40°$. Where measurements at this temperature difference were not available, the value of $\frac{R_1}{T_1}$ was found by an interpolation (or in some few cases by a slight extrapolation). This can be done with sufficient exactitude, since $\frac{R_1}{T_1}$ only varies slightly with T_1. The accuracy of the measurements is essentially less for the measurements in atmospheric air than for the measurements in hydrogen, because the radiometer forces are very small in atmospheric air; the deflections read on the scale were all less than 3 cm.

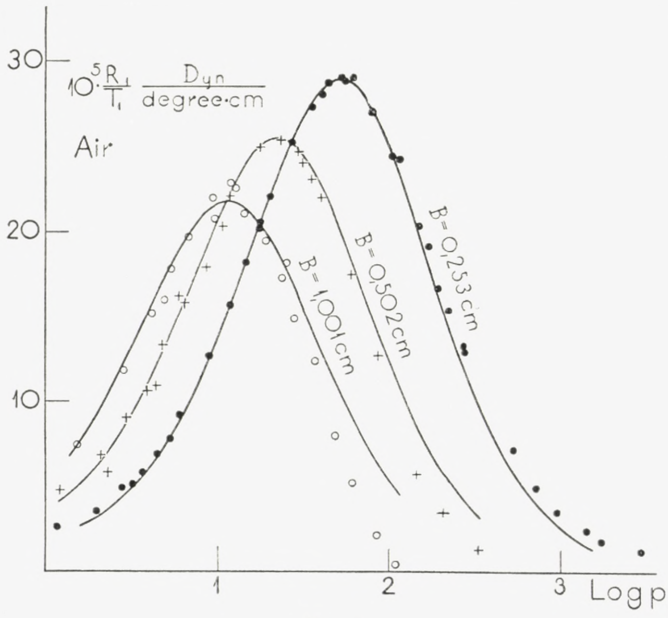
In the hydrogen measurements the deflections were as a rule 20—30 cm.

The relative accuracy of the measurements is certainly least for the measurements with the widest bands.



$10^5 \cdot \frac{R_1}{T_1}$ $\frac{Dyn}{degree \cdot cm}$
Hydrogen
300
200
100
B = 1,001 cm
B = 0,502 cn.
B = 0,253 cm
1
2
3
4
Log p

Fig. 1.



$10^5 \cdot \frac{R_1}{T_1}$ $\frac{Dyn}{degree \cdot cm}$
Air
30
20
10
B = 1,001 cm
B = 0,502 cm
B = 0,253 cm
1
2
3
Log p

Fig. 2.

This is due to the great loss of heat from the wide bands, owing to which a fairly stable condition of heat conduction cannot be achieved, and for that reason there is some uncertainty in the determination both of the radiometer force and of the temperature of the band. In addition to accidental errors there are some of a systematic nature, which are of special importance for the measurements in atmospheric air, but are without any great influence on the measurements in hydrogen. The first of these errors is due to the imperfect vacuum. From various

$10^5 \cdot \frac{R_1}{T_1}$ in hydrogen, $B = 0{,}253$ cm.

p	50°	92°	143°
14390	2,1	2,1	2,4
8590	3,9	4,4	5,1
4654	8,9	10,3	12,3
2872	16,5	19,4	22,8
1715	32,8	37,9	44,0
1235	51,5	55,1	63,8
	29°	50°	92°
779	79,0	86,2	95,4
578	107,4	115,4	125,3
429	138	148	154
5,24	39,9	39,8	38,8
10,46	66,5	66,3	62,1
15,64	91,0	91,1	88,0
20,81	116,7	116,4	112,4
26,0	134	136	136
36,2	169	169	169
38,2	183	182	
73,9	232	231	

p	19°	29°	50°
318	161	172	182
236	188	201	212
	19°	29°	40°
107,7	231	238	240
139,7	226	231	233
199,0	204	209	211
64,6	225	225	226
125,9	231	234	236
183,8	214	215	216
347	149	152	154
499	112,5	115,6	118,1
814	68,1	70,6	72,7
1408	36,4	37,9	38,9
3,08			21,8
6,09			41,7
9,07			59,9
14,96			92,2
20,84			122,2
26,6			141,2
32,4			158
41,0			180
49,6			195

$$10^5 \cdot \frac{R_1}{T_1} \text{ in hydrogen, } B = 0{,}502 \text{ cm.}$$

p	32°	62°	p	32°	43°	p	32°	43°
2,39	27,5	31,7	45,9	273	265	94,2	245	241
4,72	62,3	59,3	89,2	270	262	122,4	219	219
7,04	89,4	89,6	130,4	223	223	345	106	107
9,35	116,0	112,6	169,3	191	195	676	45,9	47,6
11,63	136,5	133,6	102	257	253	993	25,7	26,8
7,49	89,8	93,5	199	174,5	175,5	1298	16,9	17,5
14,89	163,7	157,7	292	123,3	123,4	1873	9,5	9,2
22,28	211	201	379	92,3	93,3		32°	47°
			463	73,3	73,5			
			33,1	242	245	29,6	232	232
			64,5	263	258	36,9	255	248
						44,2	264	264

$$10^5 \cdot \frac{R_1}{T_1} \text{ in hydrogen, } B = 1{,}001 \text{ cm.}$$

p	33°	43°	p	33°	43°	p	43°
5,46	118	110	227	101	102	0,98	25
10,82	199	183	169	133	132	1,91	48
16,16	242	238	125	164	166	2,84	70
21,47	272	266	93	205	206	3,76	91
26,8	290	284	69	246	241	4,67	113
32,1	296	292	51	279	275	5,59	128
17,9	267	271	263	95	95	6,49	145
29,2	306	301	514	37	37	7,40	161
39,8	310	306	755	20	20	8,30	175
49,9	301	297	1260	8	8		
59,5	288	285	1747	4	4		

causes, amongst others the giving off of adsorbed air, the apparatus could not be exhausted to any high vacuum; owing to the residue of air a final value $R_{1\,\mathrm{vac}}$ of the radiometer force is found by measuring in »vacuum«.

$10^5 \cdot \frac{R_1}{T_1}$ in atmospheric air, B = 0,253 cm.

p	40°	50°	p	40°	50°	p	40°	50°	p	40°	50°
3,26	5,1	5,3	34,6	27,4	27,9	53,2	28,9	28,6	2950	1,2	1,1
6,10	9,2	9,3	43,1	28,8	29,2	103,4	24,5	25,8			
8,93	12,7	12,7	51,6	29,1	29,2	151	20,4	20,6	1,18	2,6	2,5
11,75	15,7	16,0	60,1	29,1	29,9	196	16,7	17,5	2,02	3,5	3,7
14,56	18,2	18,4	40,0	28,1	28,2	273	13,4	13,7	2,86	4,9	4,8
17,35	20,2	20,8	77,8	27,1	27,5	534	7,2	7,2	3,69	5,8	5,8
20,04	22,1	22,3	113,4	24,3	24,3	729	4,9	5,0	4,52	6,9	6,9
8,89	12,8	13,0	170,7	19,2	19,7	1436	2,4	2,5	5,35	7,8	8,0
17,42	20,6	20,7	226	15,4	16,1	965	3,5	3,6			
26,1	25,3	25,5	279	13,0	13,2	1754	1,8	1,7			

$10^5 \cdot \frac{R_1}{T_1}$ in atmospheric air, B = 0,502 cm.

p	31°	61°	p	31°	61°	p	31°	61°	p	31°	61°
5,99	16,1	16,3	40,0	21,4	23,3	339	1,3	1,4	1,24	4,6	4,8
11,73	22,0	22,2	30,8	23,2	26,0	2,34	4,8	7,8	2,14	6,8	6,8
17,44	25,0	25,1	59,7	16,6	19,3	4,42	10,2	12,3	3,04	9,0	9,1
23,13	24,8	26,5	87,0	11,7	14,7	6,48	16,0	15,4	3,94	10,5	10,8
28,8	24,0	26,1	147,7	4,9	7,3	8,54	17,8	18,1	4,83	13,4	13,0
34,4	21,9	25,4	205	3,2	4,0	10,59	20,1	20,8			

$10^5 \cdot \frac{R_1}{T_1}$ in atmospheric air, B = 1,001 cm.

p	33°	43°	p	33°	43°	p	33°	43°	p	33°	43°
4,94	15,9	16,1	27,9	14,1	15,2	61,4	4,8	5,4	4,16	14,9	15,3
9,58	20,5	20,9	12,6	22,2	22,8	85,5	1,7	2,2	5,45	17,5	17,9
14,20	20,7	21,3	24,9	17,4	18,5	109	0,2	0,5	6,74	19,3	19,9
18,8	18,9	19,7	37,1	11,7	12,7	1,57	7,1	7,5	9,28	21,6	22,2
23,4	16,7	17,5	49,3	7,4	8,3	2,87	11,6	11,9	11,82	22,2	23,2

As it is not known of what gases the residual air consists the residual pressure p_r cannot be calculated from the measured $R_{1\,\mathrm{vac}}$; but its air- or hydrogen-equivalent can be calculated, i. e. the pressure p_L of atmospheric air, or p_H of hydrogen, which would give the same radiometer force.

Professor Knudsen's formula for the radiometer force at low pressures[1] may be written

$$\frac{R_1}{T_1} = \frac{1}{4} \cdot \frac{p}{T_0} \cdot B \cdot (a' - a'') \tag{1}$$

for small values of T_1. From this is found

$$p_L = \frac{R_{1\,\mathrm{vac}}}{T_1} \cdot \frac{4 \cdot T_0}{B\,(a' - a'')_L},$$

where $(a' - a'')_L$ denotes the difference between the coefficients of accomodation for the two sides of the band proportionate to air. It is here put $= 0{,}08$ and need not be known with any great exactitude.

If after an exhaustion of the apparatus $R_{1\,\mathrm{vac}}$ has been measured, and then by means of the pipette system as much dry atmospheric air has been introduced as gives the pressure p^1, the corrected value for the pressure in the apparatus is

$$p = p^1 + p_L.$$

The tabulated air pressures are all corrected in this way. The magnitude of the correction p_L in all cases lies between 0,2 and 0,4 Dyn/cm^2 and is thus only of importance for the measurements at low pressures.

For the hydrogen measurements the corresponding cor-

[1] V. S. math. fys. Medd. XI, 1. p. 23.

rections of the pressure p_H are much smaller, at most 0,07 Dyn/cm^2; in their calculation $(a' - a'')_H = 0{,}32$ was employed.

Another error relates to the measurements of the deflection.

The measurements were carried out in the following way. The heating current i_1 was first sent through the platinum band in an arbitrary direction and the deflection α thus produced was read. Then the current i_1 was reversed and α again read, and the mean value of the two readings of α was used.

The object of this is to eliminate the influence of the earth's magnetic field, which was, however, reduced as much as possible by the appropriate placing of a permanent magnet.

But the method fails in the case of the magnetic field which is due to the current i_1 in the conducting wires to the swinging system; the effect of the latter may become appreciable for large values of i_1.

The effect of the magnetic field of the conducting wires is evidently independent of the direction of the current, and with the arrangement employed it was opposite to the direction of the radiometer force.

For large values of i_1, which occur especially in measurements with the broad bands and at high pressures of air and hydrogen, α will thus be too small.

The correction for α is determined by the introduction of so high an air pressure in the apparatus that the radiometer force may be regarded as negligible, after which the negative deflection α_m is measured which is produced when a current i_1 of suitable strength is sent through the swinging system.

Since α_m must be proportional to i_1^2 we may put $\alpha_m = k \cdot i_1^2$, where k is a constant.

The mean value of several measurements at $^1/_2$—1 cm air pressure was found to be $k = 0{,}035$. This value was in satisfactory agreement with an estimated value of the effect of the field.

The positive correction $k \cdot i^2$ should, then, be added to the read values of the deflection α. The corrections are generally small and are only of importance for the measurements at the highest pressures, and more especially for the measurements with atmospheric air where the radiometer forces are small.

Discussion.

From figures 1 and 2, where the ordinate scale in fig. 1 is 10 times smaller than that in fig. 2, it will appear that the quantity $\frac{R_1}{T_1}$ in hydrogen is generally about 10 times larger than in atmospheric air.

Further $\frac{R_1}{T_1}$ at low pressures of hydrogen is fairly accurately proportional to the width of the band B, which agrees with Professor Knudsen's theory and with previous experience.

At low pressures of atmospheric air $\frac{R_1}{T_1}$ does not seem to increase as rapidly as B; but this is presumably due to the fact that it has not been possible to make the measurements at sufficiently low pressures.

At high pressures $\frac{R_1}{T_1}$ decreases rapidly with increasing B, and within a certain range of pressures is very nearly inversely proportional to B. Here, then, there is no question of an edge effect.

The maximum value $\frac{R_{1m}}{T_1}$ is only slightly dependent on B; when B increases from 0,25 to 1 cm, i. e. becomes 4 times as great, $\frac{R_{1m}}{T_{1\,\text{hydrogen}}}$ rises in the ratio $\frac{304}{247} = 1{,}24$, and $\frac{R_{1m}}{T_{1\,\text{air}}}$ decreases in the ratio $\frac{22}{29} = 0{,}76$.

The pressure p_m at which the maximum radiometer force $\frac{R_{1m}}{T_1}$ occurs is about inversely proportional to B.

The following table gives the graphically determined values of $\frac{R_{1m}}{T_1}$ and p_m.

	B	$10^5 \cdot \frac{R_{1m}}{T_1}$	p_m	$a' - a''$	c
Hydrogen..	0,253	247	107	0,314	0,401
	0,502	272	59	0,316	0,368
	1,001	304	33	0,316	0,329
Air.........	0,253	29,2	51	0,078	0,456
	0,502	25,8	22,4	0,079	0,532
	1,001	21,9	11,5	0,065	0,509

An empirical formula of the results is given in the expression

$$(2) \qquad \frac{R_1}{T_1} = \frac{\frac{1}{4}(a' - a'') \cdot \frac{B}{T_0} \cdot p}{1 + c \cdot \frac{B}{\lambda_1} \cdot p + \left(c \cdot \frac{B}{\lambda_1} \cdot p\right)^2 + \left(\frac{1}{3} c \cdot \frac{B}{\lambda_1} \cdot p\right)^3}$$

which for low pressures agrees with Professor KNUDSEN's above-mentioned formula, and which is obtained from a previously employed interpolation formula by regarding the container as being infinitely great[1]. Owing to dimensional considerations it was expected that the constant c, which,

[1] V. S. math.-fys. Medd. XI 9, p. 16.

like $a' - a''$, is a pure number, should be independent of B and λ_1, when the container is of infinite dimensions.

The constants $a' - a''$ and c are most readily determined by means of the values of $\frac{R_{1m}}{T_1}$ and p_m; by differentiation of (2) we find

$$a' - a'' = \frac{11{,}73\ R_{1m} \cdot T_0}{B \cdot p_m \cdot T_1} \quad \text{and} \quad c = 0{,}9660 \cdot \frac{\lambda_1}{B \cdot p_m}.$$

In the table of $\frac{R_{1m}}{T}$ and p_m the values thus calculated of $a' - a''$ and c, are also given, λ_1 the mean free path at the pressure 1 Dyn/cm^2 being put $= 11{,}23$ cm in hydrogen and $= 6{,}08$ cm in air.

As will be seen, the values of c vary somewhat both with B and λ_1.

This variation may partly be due to the fact that the above-mentioned condition of the infinite dimensions of the container is not fulfilled (the diameter of the container was c. 23 cm); but the fact that, with increasing values of B, c decreases in hydrogen and increases in atmospheric air can hardly be explained by the geometric conditions alone.

The full-drawn curves in figs. 1 and 2 are calculated from formula (2) with the given values of the constants. T_0 is put $= 293°$.

As will appear, the measurements in hydrogen are presented with good approximation, whereas the curves do not agree so well with the measurements in air, notably at high pressures. But on account of the rather marked inaccuracy of these measurements no further inferences should be drawn from this lack of agreement.

In order to investigate the effect of changes in the dimensions of the container, a series of measurements were made of the radiometer force on a platinum band 1 cm wide which was surrounded by a vertical brass tube, the transverse section of which was rectangular; its internal dimensions were 3,2 cm and 1,48 cm, so that the distance from the edges of the platinum band to the wall of the tube was only about 2,4 mm, i. e. $^1/_4$ of the width of the band.

Fig. 3 shows a horizontal section of the apparatus. *P* is the platinum band, *M* the brass tube, and *G* the glass cylinder in which the whole apparatus is enclosed; the arrangement is otherwise exactly as previously described[1].

Fig. 3.

The platinum band was 14,83 cm long and 0,999 cm wide; it was blackened with platinum black on one side in the same way as the other platinum bands.

Opposite the middle of the band a hole was drilled through the brass tube, through which the edge of the band could be observed through a microscope.

The measurement of the radiometer force was carried out by the magnetic compensation method described by Professor Knudsen[2].

The results of the measurements are given in the table below, and are presented graphically for $T_1 = 80°$ in fig. 4, the ordinate scale of which is about 5 times smaller than the ordinate scale in fig. 1. Thus the radiometer forces are about 5 times as great.

As regards the accuracy of measurement it must be noted that it is rather small for high pressures, for the reason already mentioned (great loss of heat, no stable

[1] V. S. math.-fys. Medd. XI, 9, p. 3.
[2] V. S. math.-fys. Medd. XI, 1, p. 32—34.

$10^4 \cdot \frac{R_1}{T_1}$ in hydrogen.

p	40°	80°	120°	p	80°
50,0	99	87	86	2,75	8,4
97,4	144	119	123	5,48	16,2
142,4	151	134	135	8,21	24,6
301	132	122	125	10,92	29,2
452	109	96	111	13,62	35,3
596	90	89	95	16,31	41,1
3460	36	26	30	19,00	46,3
1950	56	40	43	21,66	50,6
964	73	58	63		
715	82	66	79	5,25	11,8
				10,49	27,5
38,2	80	75	71	15,70	39,4
73,9	111	113	107	20,9	48,3
107,7	131	130	123	26,1	56,6
139,7	135	136	131	31,2	62,9
199	167	154	140	36,3	71,0
64,6	119	114	99		
125,9	156	141	128		
183,8	153	142	131		
347	133	122	117		
499	95	100	100		
814	80	68	78		

conduction of heat); at low pressures it is appreciably greater.

The maximum values are

$$\frac{R_{1m}}{T_1} = 147 \cdot 10^{-4} \frac{\text{Dyn}}{\text{cm degree}}, \; p_m = 166 \frac{\text{Dyn}}{\text{cm}^2}.$$

The results may in this case be represented by a symmetrical radiometer curve

$$\frac{R_1}{T_1} = \frac{\frac{1}{4}(a' - a'') \frac{B}{T_0} \cdot p}{1 + bp + (bp)^2}, \tag{4}$$

which is obtained from (2) by omitting the 3rd degree term of the denominator and putting $c \cdot \frac{B}{\lambda_1} = b$. The constants are determined by $a' - a'' = \frac{12 \cdot R_{1m} \cdot T_0}{B \cdot p_m \cdot T_1}$ and $b = \frac{1}{p_m}$, which gives $a' - a'' = 0{,}314$, $b = \frac{1}{166}$.

The full-drawn curve in fig. 4 was calculated from (4)

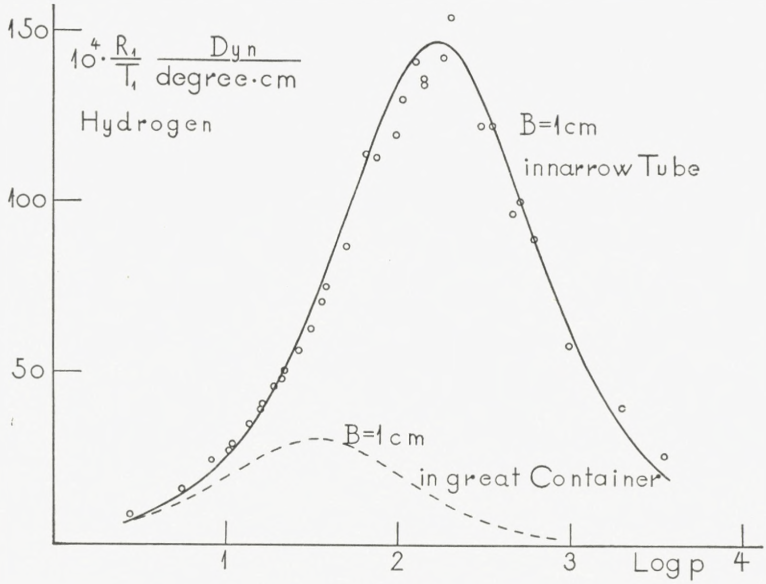


Fig. 4.

with these constants. In the same figure the radiometer curve for the widest band $B = 1{,}001$ cm in the large container is given (in a dotted line) for comparison.

The author desires to thank Professor Knudsen for excellent working conditions and for his constant interest in the work, and likewise tenders thanks to the trustees of »Emil Herborgs Legat« for a grant of funds.

Færdig fra Trykkeriet d. 29. Juni 1935.